Preface

In the 1960s there was considerable ferment in mathematics curriculum and instruction. Although public attention was focused on the more visible attempts at program revision, we are aware two decades later that change was more apparent than real.

In the 1970s the concern of the public was directed toward problems evidenced almost exclusively in test scores. Schools have responded to this concern in a variety of ways, but a clear-cut and carefully reasoned sense of direction that looks toward the future has been lacking.

We recognize as valid and legitimate the role of public opinion in the determination of educational goals. But this philosophy is predicated on a well-informed public. Thus, the National Council of Teachers of Mathematics, as an organization of professional educators, has a special obligation to present its responsible and knowledgeable viewpoint of the directions mathematics programs should be taking in the 1980s.

These recommendations represent both realism and responsibility. They are realistic in their attention to hard data. We are fortunate to have more information about mathematics classroom practice than we have ever had. This useful information comes principally from a series of studies funded by the National Science Foundation and from two mathematics assessments of the National Assessment of Educational Progress.

The recommendations are responsible to the profession and to the public since they represent a very broad base of belief about objectives and priorities. The Council, funded by the National Science Foundation, conducted an extensive survey of the opinions of many sectors of society, both lay and professional. The project was called Priorities in School Mathematics (PRISM).

Such opinion surveys do not in themselves generate recommendations, but a professional organization, if it is to be responsible, must give them serious consideration as it develops its best-considered advice to society concerning future directions for educational programs.

These recommendations are not the end of our efforts but a beginning. They
represent an agenda for a decade of action, and we call on all interested persons and groups to join us in a massive cooperative effort toward better mathematics education for all our youth.

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An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;

2. basic skills in mathematics be defined to encompass more than computational facility;

3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;

5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;

6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;

7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;

8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.
Recommendation 1
PROBLEM SOLVING MUST BE THE FOCUS OF SCHOOL MATHEMATICS IN THE 1980s

The development of problem-solving ability should direct the efforts of mathematics educators through the next decade. Performance in problem solving will measure the effectiveness of our personal and national possession of mathematical competence.

Problem solving encompasses a multitude of routine and commonplace as well as nonroutine functions considered to be essential to the day-to-day living of every citizen. But it must also prepare individuals to deal with the special problems they will face in their individual careers.

Problem solving involves applying mathematics to the real world, serving the theory and practice of current and emerging sciences, and resolving issues that extend the frontiers of the mathematical sciences themselves.

This recommendation should not be interpreted to mean that the mathematics to be taught is solely a function of the particular mathematics needed at a given time to solve a given problem. Structural unity and the interrelationships of the whole should not be sacrificed.

True problem-solving power requires a wide repertoire of knowledge, not only of particular skills and concepts but also of the relationships among them and the fundamental principles that unify them. Each problem cannot be treated as an isolated example. This recommendation looks toward the need to solve problems in an uncertain future as well as here and now. As such, mathematics needs to be taught as mathematics, not as an adjunct to its fields of application. This demands a continuing attention to its internal cohesiveness and organizing principles as well as to its uses.

Recommended Actions

1.1 *The mathematics curriculum should be organized around problem solving.*

- The current organization of the curriculum emphasizes component computational skills apart from their application. These skills are necessary tools but should not determine the scope and sequence of the curriculum. The need of the student to deal with the personal, professional, and daily experiences of life requires a curriculum that emphasizes the selection and use of these skills in unexpected, unplanned settings.

- Mathematics programs of the 1980s must be designed to equip students with the mathematical methods that support the full range of problem solving, including—
  - the traditional concepts and techniques of computation and applications of mathematics to solve real-world problems, the rational and real number systems, the notion of function, the use of mathematical symbolism to describe real-world relationships, the use of deductive and inductive reasoning to draw conclusions about such relationships, and the geometrical notions so useful in representing them;
  - methods of gathering, organizing, and interpreting information, drawing and testing inferences from data, and communicating results;
  - the use of the problem-solving capacities of computers to extend traditional problem-solving approaches and to implement new strategies of interaction and simulation;
  - the use of imagery, visualization, and spatial concepts.

- Mathematics programs should give students experience in the application of mathematics, in selecting and matching strategies to the situation at hand. Students must learn to—
  - formulate key questions;
  - analyze and conceptualize problems;
  - define the problem and the goal;
  - discover patterns and similarities;
  - seek out appropriate data;
  - experiment;
  - transfer skills and strategies to new situations;
  - draw on background knowledge to apply mathematics.

- Fundamental to the development of problem-solving ability is an open mind, an attitude of curiosity and exploration, the willingness to probe, to try, to make intelligent guesses.

- The curriculum should maintain a balance between attention to the applications of mathematics and to fundamental concepts.

1.2 The definition and language of problem solving in mathematics should be developed and expanded to include a broad range of strategies, processes, and modes of presentation that encompass the full potential of mathematical applications.

- Computational activities in isolation from a context of application should not be labeled “problem solving.”

- The definition of problem solving should not be limited to the conventional “word problem” mode.

- As new technology makes it possible, problems should be presented in more natural settings or in simulations of realistic conditions.

- Educators should give priority to the identification and analysis of specific problem-solving strategies.

- Educators should develop and disseminate examples of “good problems” and strategies and suggest the scope of problem-solving activities for each school level.
1.3 Mathematics teachers should create classroom environments in which problem solving can flourish.

- Students should be encouraged to question, experiment, estimate, explore, and suggest explanations. Problem solving, which is essentially a creative activity, cannot be built exclusively on routines, recipes, and formulas.
- The mathematics teacher should assist the student to read and understand problems presented in written form, to hear and understand problems presented orally, and to communicate about problems in a variety of modes and media.
- The mathematics curriculum should provide opportunities for the student to confront problem situations in a greater variety of forms than the traditional verbal formats alone; for example, presentation through activities, graphic models, observation of phenomena, schematic diagrams, simulation of realistic situations, and interaction with computer programs.

1.4 Appropriate curricular materials to teach problem solving should be developed for all grade levels.

- Most current materials strongly emphasize an algorithmic approach to the learning of mathematics, and as such they are inadequate to support or implement fully a problem-solving approach. Present textbook problems tend to be easily categorized and stylized and often bear little resemblance to highly diversified, real-life problems. They do not permit the full range of strategies and abilities actually demanded in realistic problem contexts.
- The potential of computing technology for increasing problem-solving ability should be explored and exploited by the development of creative and imaginative software.

1.5 Mathematics programs of the 1980s should involve students in problem solving by presenting applications at all grade levels.

- Applications should be presented that use the student’s growing and changing repertoire of basic skills to solve a multitude of routine and commonplace problems essential to the day-to-day living of every citizen.
- Applications of mathematics to other disciplines such as the social sciences, business, engineering, and the natural sciences should be presented.
- The enormous versatility of mathematics should be illustrated by presenting a diversified a collection of applications as possible at the given grade level.
- At the college level, courses in mathematics and the mathematical sciences should give prospective teachers experiences that develop their capacities in modeling and problem solving.

1.6 Researchers and funding agencies should give priority in the 1980s to investigations into the nature of problem solving and to effective ways to develop problem solvers.

- Support should be provided for—
  - the analysis of effective strategies;
  - the identification of effective techniques for teaching;
  - new programs aimed at preparing teachers for teaching problem-solving skills;
  - investigations of attitudes related to problem-solving skills;
  - the development of good prototype material for teaching the skills of problem solving, using all media.

Recommendation 2

THE CONCEPT OF BASIC SKILLS IN MATHEMATICS MUST ENCOMPASS MORE THAN COMPUTATIONAL FACILITY

There must be an acceptance of the full spectrum of basic skills and recognition that there is a wide variety of such skills beyond the mere computational if we are to design a basic skills component of the curriculum that enhances rather than undermines education.

We recognize as valid and genuine the concern expressed by many segments of society that basic skills be a part of the education of every child. However, the full scope of what is basic must include those things that are essential to meaningful and productive citizenship, both immediate and future.

The agreement among parents, educators, and mathematicians on the need for teaching basic skills with greater effectiveness unfortunately does not yet extend to a common understanding and acceptance of exactly what these basic skills should comprise.

Some groups narrowly limit them to routine computation at the expense of understanding, applications, and problem solving. This would leave little hope of developing the functionally competent student that all desire.

It must also be recognized that individual capacities, interests, and future directions might call for different emphases and different selections in matching basic skills to individual needs.

The time and energy that teachers and programs should be devoting to building beyond minimal foundations are sometimes skirted, being considered risky deviations from the minimal targets on which educators believe they will be judged. There is great pressure today to use all such time, energy, and re-
sources on overkill in the minimal target areas even though little added productivity may be achieved.

Rather than fostering a return to some acceptable common threshold of performance, the back-to-basics movement tends to place a low ceiling on mathematical competence—and this at the onset of an era in which daily life will be more deeply permeated by multiple and diverse uses of mathematics than ever before. Under these circumstances, even if improvement in rote computation takes place, a citizen who cannot analyze real-life situations to the point of recognizing what computations must be made to solve real-life problems has not entered the mainstream of functional citizenship.

It is dangerous to assume that skills from one era will suffice for another. Skills are tools. Their importance rests in the needs of the times. Skills once considered essential become obsolete, and this is likely to increase in pace and scope as advances in technology revolutionize our individual, social, and economic lives. Necessary new skills arise from the dimensions of the mathematics pertinent to an age of population explosion, space exploration, economic and fiscal complexity, and microelectronic wonders. Time and space for including these new skills in the curriculum must be purchased by eliminating the obsolete.

Insisting that students become highly facile in paper-and-pencil computations such as $3841 \times 937$ or $72509 + 293$ is time-consuming and costly. For most students, much of a full year of instruction in mathematics is spent on the division of whole numbers—a massive investment with increasingly limited productive return. A small fraction of that time is spent on the skills of problem analysis and interpretation, which enable students to identify and set up the computations needed. For most complex problems, using the calculator for rapid and accurate computation makes a far greater contribution to functional competence in daily life.

Common sense should dictate a reasonable balance among mental facility with simple basic computations, paper-and-pencil algorithms for simple problems done easily and rapidly, and the use of a calculator for more complex problems or those where problem analysis is the goal and cumbersome calculating is a limiting distraction.

Reasonable standards of time-effectiveness and cost-effectiveness should be applied to the use of instructional time, where the criterion is the productive applicability of the learned technique to real-life problems.

Professional knowledge of future trends, industrial, financial, engineering, and scientific need, and the demands of daily life are all better arbiters of what is currently essential and what has become obsolete than our nostalgia as parents or teachers.

**Recommended Actions**

2.1 The full scope of what is basic should contain at least the ten basic skill areas identified by the National Council of Supervisors of Mathematics's "Position Paper on Basic Skills." These areas are problem solving; applying mathematics in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy.

2.2 The identification of basic skills in mathematics is a dynamic process and should be continually updated to reflect new and changing needs.

2.3 Changes in the priorities and emphases in the instructional program should be made in order to reflect the expanded concept of basic skills.

- There should be increased emphasis on such activities as—
  - locating and processing quantitative information;
  - collecting data;
  - organizing and presenting data;
  - interpreting data;
  - drawing inferences and predicting from data;
  - estimating measures;
  - measuring using appropriate tools;
  - mentally estimating results of calculations;
  - calculating with numbers rounded to one or two digits;
  - using technological aids to calculate;
  - using ratio and proportion to deal with rate problems in general and with percent problems in particular;
  - using imagery, maps, sketches, and diagrams as aids to visualizing and conceptualizing a problem;
  - using concrete representations and puzzles that aid in improving the perception of spatial relationships.

- There should be decreased emphasis on such activities as—
  - isolated drill with numbers apart from problem contexts;
  - performing paper-and-pencil calculations with numbers of more than two digits;
  - mastering highly specialized vocabulary not useful later either in mathematics or in daily living;
  - converting measures given in one system to corresponding measures in another system;
  - working with tables whose usefulness as aids to calculation has been supplanted by calculators and other technological aids (e.g., numerical computations with logarithms and cologs).

2.4 Teachers should incorporate estimation activities into all areas of the program on a regular and sustaining basis, in particular encouraging the
use of estimating skills to pose and select alternatives and to assess what
a reasonable answer may be.

2.5 Teachers should provide ample opportunities for students to learn
communication skills in mathematics. They should systematically guide stu-
dents to read mathematics and to talk about it with clarity.

2.6 The higher-order mental processes of logical reasoning, information pro-
cessing, and decision making should be considered basic to the appli-
cation of mathematics. Mathematics curricula and teachers should set as
objectives the development of logical processes, concepts, and language,
including—

—the identification of likenesses and differences leading to classifi-
cation;
—understanding, making, and applying definitions;
—the development of a feeling for informal proof including counter-
examples and generalizations;
—precise use of such language as at least, at most, either-or, both-and,
and if-then.

Recommendation 3

MATHEMATICS PROGRAMS MUST TAKE FULL
ADVANTAGE OF THE POWER OF CALCULATORS
AND COMPUTERS AT ALL GRADE LEVELS

Beyond an acquaintance with the role of computers and calculators in
society, most students must obtain a working knowledge of how to use them,
including the ways in which one communicates with each and commands their
services in problem solving.

The availability of computing aids, including computers and calculators,
requires a reexamination of the computational skills needed by every citizen.
Some of these computational skills will no longer retain their same impor-
tance, whereas others will become more important.

It is recognized that a significant portion of instruction in the early grades
must be devoted to the direct acquisition of number concepts and skills with-
out the use of calculators. However, when the burden of lengthy computa-
tions outweighs the educational contribution of the process, the calculator should
become readily available.

With the increasing availability of microcomputers at decreasing costs, it is
imperative that schools play an active part in preparing students of the 1980s
to live in a world in which more and more functions are being performed by
computers.

Recommended Actions

3.1 All students should have access to calculators and increasingly to com-
puters throughout their school mathematics program.

• Schools should provide calculators and computers for use in ele-
mentary and secondary school classrooms.
• Schools should provide budgets sufficient for calculator and com-
puter maintenance and replacement costs.

3.2 The use of electronic tools such as calculators and computers should be
integrated into the core mathematics curriculum.

• Calculators should be available for appropriate use in all mathe-
matics classrooms, and instructional objectives should include the
ability to determine sensible and appropriate uses.
• Calculators and computers should be used in imaginative ways for
exploring, discovering, and developing mathematical concepts and
not merely for checking computational values or for drill and
practice.
• Teachers should ensure in their classroom management that the use
of computers by individual students in isolated activity does not
replace the critical classroom interaction of students with peers and
teacher. The healthy give-and-take of group work and discussion,
which promotes values of communication, cooperation, empathy,
mutual respect, and much of cognitive development, remains es-
sential.

3.3 Curriculum materials that integrate and require the use of the calculator and
computer in diverse and imaginative ways should be developed and
made available.

• Schools should insist that materials truly take full advantage of the
immense and vastly diverse potential of the new media. In particu-
lar, developers of software should be cautioned that just to use con-
ventional material and techniques newly translated to the medium of
the computer will not suffice.
• Educators should take care to choose software that fits the goals or
objectives of the program and not twist the goals and developmental
sequence to fit the technology and available software.

3.4 A computer literacy course, familiarizing the student with the role and
impact of the computer, should be a part of the general education of
every student.

• In cooperation with schools and professional teacher organizations,
funding agencies should support the development of courses in com-
puter literacy for both junior and senior high school levels.
3.5 All mathematics teachers should acquire computer literacy either through preservice programs or through in-service programs funded by school districts in order to deal with the impact of computers on their own lives and to keep pace with the inevitable sophistication their students will achieve.

- Colleges should provide courses for both preservice and in-service education in computer literacy, programming, and instructional uses of calculators and computers.
- Professional organizations should provide information through their various media, conferences, workshops, and seminars to aid in the in-service education of teachers in uses of the calculator and computer.

3.6 Secondary school computer courses should be designed to provide the necessary background for advanced work in computer science.

- Curriculum design should provide the required foundation for those students who will be involved in careers that increasingly demand advanced computing skills and applications of computing and for those students who will go on to deeper study in frontier fields of computer development.

3.7 School administrators and teachers should initiate interaction with the home to achieve maximum benefit to the student from the coordinated home and school use of computers and calculators.

- Criteria should be developed to assist parents and school personnel in their selection of home/school computing hardware.
- Professional organizations of teachers, mathematicians, and computer scientists should develop guidelines to aid schools, teachers, and parents in the selection of educational software.
- The uses of technological devices such as calculators, computers, video disks, and electronic games in the home and other out-of-school places should be anticipated. Programs should be planned that will encourage the positive and educationally beneficial use of these devices.
- As home computers come into wider use, homework should be assigned that can take advantage of their potential in problem solving.

3.8 Educational users of electronic technology should demand a dual responsibility from manufacturers: the development of good software to promote the problem-solving abilities of the student and, eventually, the standardization and compatibility of hardware.

3.9 Provisions should be made by educational institutions and agencies to help in the necessary task of educating society's adults in computer literacy and programming.

3.10 Teachers of other school subjects in which mathematics is applied should make appropriate use of calculators and computers in their instructional programs.

3.11 Teacher education programs for all levels of mathematics should include computer literacy, experience with computer programming, and the study of ways to make the most effective use of computers and calculators in instruction.

3.12 Certification standards should include preparation in computer literacy and the instructional uses of calculators and computers.

Recommendation 4

STRINGENT STANDARDS OF BOTH EFFECTIVENESS AND EFFICIENCY MUST BE APPLIED TO THE TEACHING OF MATHEMATICS

What is learned relative to a topic, how long it is retained, how readily it is applied—all these depend on the learning process the students pass through and how effectively they are engaged in that process. It is fruitless to consider topics taught apart from the way learners meet these topics.

Instructional time is a precious commodity. It must be spent wisely. Learning is a product of both the time engaged in a learning task and the quality of that engagement. Teachers must employ the most effective and efficient techniques at their command. They must apportion instructional time according to the importance of the topic, recognizing that the value of a skill or knowledge is subject to change over time.

Modern technology and educational theory and research have made accessible to today's teacher approaches, materials, and strategies that were not previously available. Teachers at all levels must learn to use this enriched variety of instructional techniques, materials, and resources to teach mathematics more effectively.

Recommended Actions

4.1 The major emphasis on problem solving in the curriculum must be accommodated by a reprogramming of the use of time in the classroom.

- Priority in classroom time should be devoted to involving students in meaningful problem-solving activities. Explanation, practice, and directive teaching are important but should not diminish the time necessary to achieve this priority. Requiring complete mastery of skills before allowing participation in challenging problem solving is counterproductive.
The time spent on mathematics in elementary school programs should be increased. Higher-order skills in problem solving require more time to learn than the lower-order, narrowly mechanistic skills.

The extent of teacher and student time devoted to certain traditional skill areas should be reduced to make room for newly emerging objectives.

There are certain algorithmic skills (e.g., long division with multipledigit divisors) that require a great expenditure of classroom time. A strict standard of time-effectiveness and cost-effectiveness should be applied to determine whether actual use of that technique in life outside school justifies this much expenditure of effort and time. The use of calculators has radically reduced the demand for some paperand-pencil techniques.

Teachers should learn effective techniques of classroom management to assess and achieve the optimal time on a task.

4.2 School administrators and parents must support the teacher’s efforts to engage students more effectively in learning tasks.

- Local administrators should ensure uninterrupted time for the teacher to carry out the instructional program.
- Parents and administrators must support the authority of the teacher to require that students be productively engaged in learning during their class time. They should exercise reasonable sanctions against students who do not respect that authority.
- Parents and administrators should expect that teachers will assign a reasonable amount of meaningful homework to extend productively the time students are engaged in the study of mathematics. Both should use their influence to increase the likelihood that students will complete the homework as assigned.
- School budgets should provide for a range of instructional resources adequate to support a wide variety of teaching strategies.

4.3 Teachers should use diverse instructional strategies, materials, and resources, such as—

- individual or small-group work as well as large-group work;
- well-planned use of media, such as overhead projectors, videotapes, video disks, audio/video cassettes, computers, films, slides, television;
- the provision of situations that provide discovery and inquiry as well as basic drill;
- the use of manipulatives, where suited, to illustrate or develop a concept or skill;
- the inclusion of cyclic review of past topics (contents, skills, and ideas previously taught);
- the use of materials and references outside the classroom, such as visiting museums, using the library, visiting businesses or industries, visiting computer centers, making home television assignments.

4.4 School districts, local and state or provincial officials, manufacturers, and publishers should take a bolder and more imaginative approach to selecting and producing educational hardware and software in order to provide for a curriculum that emphasizes problem solving.

- Unnecessarily restrictive conditions set by school districts, state and provincial agencies, outdated ground rules of publishing houses, conservative status quo editorial policies, and the like should give way to greater openness and willingness to address future needs.

Recommendation 5

THE SUCCESS OF MATHEMATICS PROGRAMS AND STUDENT LEARNING MUST BE EVALUATED BY A WIDER RANGE OF MEASURES THAN CONVENTIONAL TESTING

The first purpose of meaningful evaluation in school mathematics should be the improvement of learning programs, teaching, and materials. Educators must evaluate to have information for sound decisions, to be accountable to their public, and to know how well they are doing. Evaluation is a part of mathematics teaching, and hence mathematics educators should be centrally involved in the evaluation process.

Evaluation is not limited to testing. It includes gathering data and interpreting the data. Testing is one source of data. There are many others. Today, many people use test scores as the sole index of the quality of mathematics programs or of the success of student achievement. Test scores alone should not be considered synonymous with achievement or program quality. A serious danger to the education of our youth is the increasing tendency on the part of the public to assume that the sole objective of schooling is a high test score. This is often assumed without the critical knowledge of what is being tested or whether test items fit desired goals.

The evaluation of problem-solving performance will demand new approaches to measuring. Certainly present tests are not adequate. In particular, the measuring of the use of problem-solving processes will demand innovative techniques. Evaluation of programs with problem-solving goals must be sensitive to the nature of those goals.
It is imperative that the goals of the mathematics program dictate the nature of the evaluations needed to assess program effectiveness, student learning, teacher performance, or the quality of materials. Too often the reverse is true: the tests dictate the programs, or assumptions of the evaluation plan are inconsistent with the program’s goals.

**Recommended Actions**

5.1 *The evaluation of mathematics learning should include the full range of the program’s goals, including skills, problem solving, and problem-solving processes.*

- The evaluation of the use of problem-solving processes must be given special attention by schools, teachers, researchers, test developers, and teacher educators.
- Assessment programs, such as the National Assessment and some state assessments, should continue to sample a wide variety of mathematics learning outcomes and should consider future as well as present needs and programs.
- The development of problem-solving skills should be assessed for each student over the entire K-12 school mathematics program.
- Minimal competencies should not be construed as an adequate measure of an individual’s mathematics achievement. What is minimal for all is optimal for none.

5.2 *Parents should be regularly and adequately informed and involved in the evaluation process.*

- With mutual respect (the educator for the sincere concern and valid input of the parent and the parent for the professional expertise of the educator), school administrators, teachers, and parents should cooperate in determining educational goals and the appropriate plan for evaluation.
- Non-test evaluation methods and strategies should be discussed with parents, students, and the general public.

5.3 *Teachers should become knowledgeable about, and proficient in, the use of a wide variety of evaluative techniques.*

- Preservice and in-service teacher education should provide mathematics teachers with knowledge about, and skill in, evaluation.
- Evaluation strategies that include both test and non-test techniques should be developed and disseminated to mathematics teachers both in their initial preparatory programs and in continual in-service updating.
- Teachers should improve their diagnostic skills and their ability to structure appropriate remediation.

5.4 *The evaluation of mathematics programs should be based on the program’s goals, using evaluation strategies consistent with these goals.*

- Standardized tests should be used in program evaluations only when it can be clearly demonstrated that the test matches the goals of the program.
- The results of a test designed for purposes other than program evaluation (such as the SAT) should not be interpreted as an evaluation of mathematics programs.
- Available evaluation and testing techniques should not determine the goals and objectives of the mathematics program or the emphases of classroom instructional effort.
- Test designers should give attention to the need for more options in format than the conventional multiple-choice format. An emphasis on problem solving demands more flexibility and creativity in assessment than is possible within the restrictions of most current test formats.
- Where minimal competency tests are mandated, they should be implemented with extreme caution to assure that adverse effects on the program do not result.
- Task forces involving parents, teachers, and students should be created to monitor the effects of minimal competency programs.
- Mathematics educators should be centrally involved in the development of competency or assessment programs at local, state, and provincial levels and in the monitoring of the effects of competency testing on mathematics learning.
- Longitudinal evaluation of individual problem-solving ability should be developed. The acquisition of problem-solving skills is a long-term process and should not be evaluated solely with short-term measures.
- Test scores should not be used as the sole index of success in mathematics programs.
- Accreditation of school mathematics programs should use criteria specific to the quality of the mathematics programs rather than to conditions peripheral to content and instructional goals.

5.5 *The evaluation of materials for mathematics teaching should be an essential aspect of program planning.*
Textbook materials should be judged and selected in terms of the program's goals rather than vice versa.

Instructional materials with sexist and ethnic biases should not be selected.

The selection of tests should involve a careful review by teachers as well as administrators.

Strategies for evaluating nonprint materials must be developed and used.

5.6 Mathematics teachers must undergo continuing evaluation as a necessary component in improving mathematics programs.

- Teachers should maintain an awareness of the need and the strategies for self-evaluation.
- A variety of supportive evaluation strategies, such as peer observation, supervisor observation, and videotaping, should be made available to the teacher.
- Any evaluation of mathematics teaching should be sensitive to the instructional goals and should be unique to the content, the teacher, and the class.
- If an evaluation of a teacher's effectiveness includes student performance, the measures of student performance should be consistent with insitutional goals. Such evaluation should also consider that there are external factors affecting student performance that are not amenable to teacher influence.
- Judgments of teacher competency are necessary but should be made with caution, with the realization that the validity of most existing measures rests on a shaky foundation.
- Teachers and teacher educators should be centrally involved in the development of instruments for the evaluation of teaching effectiveness.

5.7 Funding agencies should support research and evaluation of the effects of a problem-solving emphasis in the mathematics curriculum.

- The nature of problem-solving ability suggests that longitudinal studies will be most meaningful. The more typical short-term project may force a hasty and superficial treatment of programs whose objectives must be complex, interrelated, and of a long-lasting character.
mathematical ideas and tools to the solution of real-life problems. Present high school programs do not fully anticipate the many options provided by mathematics, the mathematical sciences, and computer science.

Technical and vocational training at different levels also assumes more and diverse mathematical backgrounds. For those whose formal education will end with high school, the needs of citizen and consumer for increasing mathematical sophistication dictate a collection of courses based on consumer and career needs, computer literacy, and quantitative literacy.

It is important that recommended programs permit lateral movement and not strictly "track" students, trapping them in a linear pattern that does not permit change to another path. Flexibility is vital, and the key is to keep options open as long as possible.

Since a higher level of mathematical skill and understanding will increasingly become a significant advantage in nearly all lives, then justice demands that all groups have equal access to these advantages. At present, females and some minority groups are underrepresented in mathematics courses and courses for which mathematics is prerequisite.

It is naive to suppose that just providing mathematics courses as electives will serve for equality of opportunity. All reasonable means should be employed to assure that everyone will have the foundation of mathematical learning essential to fulfilling his or her potential as a productive citizen. The currently underrepresented groups should be especially encouraged and helped.

Recognizing diversified individual interests and needs entails devising programs that are tailored for particular categories of students. Differentiated curricula must incorporate the special needs in mathematics of students with handicaps, including physical or learning difficulties. These programs will need to move away from the idea that everyone must learn the same mathematics and develop the same skills. Mathematics and mathematical ability cover a much broader range than most people realize.

In many current programs, the student who does poorly in the algorithmic skills finds progress in all aspects of mathematical development halted, since remediation is designed to concentrate solely on this deficiency. Remedial programs should identify other areas of mathematical ability—for example, spatially related skills—and concentrate attention also on the students' strengths, not solely on their deficits.

The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world.

Mathematics educators and curriculum developers should redesign the sequence of the curriculum to realize the critical process goals of problem solving, as well as content and skill goals. A clear and logical developmental sequence for process objectives from kindergarten through twelfth grade should be described and serve as an organizing framework.

In very general terms, such a sequence should proceed through stages of development, though in practice, progression should be smooth and unbroken.

In the elementary school, children, in their experiences in solving particular problems, gradually develop their higher-order mental abilities and learn particular skills and particular strategies. At this stage, strategies are primarily specific to individual problems. A foundation of skills must be laid, but skills should not be learned entirely in isolation from application even in the primary grades. There should be an interplay of skill building and application throughout.

Moving through the upper grades and into the junior high school level, the progress should be toward more generalization, more abstraction of techniques, more emphasis on similarities and patterns found in differing contexts. Strategies become not just ways to solve one type of problem but generalized and synthesized. Techniques learned in one context may be recognized as applicable to other problems. Specific attention is needed to help students make the important transition to the abstract reasoning processes.

During the seventh and eighth grades, intensive focus on problem solving should become a vehicle to exercise, confirm, and develop further all basic skills. At the same time, familiarity, competence, and confidence should be built in applying these mathematical skills to solving problems of varying difficulty and from diverse settings. At this stage, a significant skill is the ability to select strategies from a growing repertoire.

A broad range of problem-solving approaches should be explored so that the teacher can identify students' special strengths and advise them on high school options.

The ability to create strategies to attack a new problem is, at the junior high school level, simple and embryonic. It should increase in sophistication throughout high school. More flexibility and power should be achieved by more generalization and abstraction. The essential nature of techniques and strategies and their range of applicability should be emphasized as students begin to see more applications to broad categories of problem areas, as in, for example, application to other disciplines. There should be a greater effort made to coordinate the mathematics learned in mathematics classes with other subjects that use mathematics. Students who perform well in mathematics classes often fail to see any relationship in what they are doing there to the mathematical techniques employed in classes in science and other subjects.

This does not suggest that the content of high school mathematics is either dictated by or limited to the range of mathematical techniques used in other subjects. The goal is to develop a more flexible, deeper, and broader problem-solving power, and this goes significantly beyond the formulas and recipes that have been traditionally applied to familiar problems.

At the same time, the important interplay and integration of mathematics and its applications in learning should not cease because isolated course structure separates mathematics from disciplines that apply it. The principles of learning must take priority over administrative convenience. It is likely that this coordination will have to be accomplished by voluntary cooperative efforts.
on the part of mathematics teachers and the teachers of other subjects that apply mathematics, with a healthy mutual respect for the legitimate goals of both groups.

At the high school level many students can apply their problem-solving abilities not only to problems of daily life, to problems from other disciplines, but to serious mathematical problems themselves.

**Recommended Actions**

6.1 **School districts should increase the amount of time students spend in the study of mathematics.**

- At least three years of mathematics should be required in grades 9 through 12.
- The amount of time allocated to learning mathematics in elementary school should be increased. It should range from a minimum of about five hours a week in the primary grades to a minimum of about seven hours a week in the upper grades. Part of this time can be gained by a program that stresses the application of mathematical skills in other subjects, especially the sciences and the social studies.
- A clear delineation of what constitutes college-level and precollege mathematics should be made.
- Colleges should not award college credit for courses in which the level of content is that of high school mathematics. This practice encourages students not to elect mathematics in high school beyond the minimum required.

6.2 **In secondary school, the curriculum should become more flexible, permitting a greater number of options for a diversified student population.**

- Increasing high school requirements in mathematics should not result in keeping all students longer in the same traditional tracks. These recommendations cannot be met with just the two-track alternatives of either general mathematics courses or precalculus courses typical of many existing programs.
- The high school curriculum should provide differing student populations with those appropriately organized areas of mathematical competence required by their needs, talents, and future objectives, but all presented with continual attention to functional problem-solving ability.
- Algebra should be included in the program of all capable students to keep their options open.
- For many students, algebra should be delayed until a level of maturity and basic mathematical understanding permit their taking full advantage of a significant algebra course. For many, this may not be ninth grade but perhaps eleventh or even twelfth grade. Significant mathematics courses should be available to these students in ninth and tenth grades, not just the traditional general mathematics review or prealgebra courses.
- If such recommendations are followed, a course providing progress beyond junior high school but paving the way for a useful experience in consumer mathematics and later algebra needs to be developed.
- Consumer mathematics should develop a broader quantitative literacy and should consist primarily of work in informal statistics, such as organizing and interpreting quantitative information.
- All high school students should have work in computer literacy, and the hands-on use of computers, and the applications of computers where possible and appropriate throughout their mathematics programs.
- All students who plan to continue their study of mathematics beyond high school or to use it extensively in technical work or training should be enrolled in mathematics courses throughout their last high school year.

6.3 **Mathematics educators and college mathematicians should reevaluate the role of calculus in the differentiated mathematics programs.**

- Emerging programs that prepare users of mathematics in nontraditional areas of application may no longer demand the centrality of calculus that has traditionally been demanded for all students. (The Mathematical Association of America's PRIME 80 conference raised questions about the role of calculus as the eventual touchstone that dictated all college preparatory mathematics.)
- In light of this reevaluation, colleges and high schools should reexamine the concept of advanced placement in mathematics. For some students, though fully capable, an advanced placement program restricted to calculus placement may not be the optimal alternative. If advanced placement in mathematics is encouraged, it should be a broader concept that includes options in other branches of the mathematical sciences.

6.4 **The curriculum that stresses problem solving must pay special heed to the developmental sequence best suited to achieving process goals, not just content goals.**

- From the earliest years, the basic mathematical tools should be acquired within the framework of usage and application, however simple the examples may be at the beginning levels.
- Since there are usually multiple approaches to all but the most trivial problems, not all students should be expected to proceed in the same way. Value should be placed on a thoughtful and productive approach, not solely on a single correct answer.
Since elementary school children differ widely in maturation and intellectual development, the teacher should be prepared to value and reward different contributions made by different students to the solution of a common problem.

Team efforts in problem solving should be commonplace in the elementary school classroom.

At middle school and junior high school levels, the focus of the curriculum should be on more formal and more general problem-solving approaches and strategies themselves.

At middle school and junior high school, instruction should stress the ability to apply techniques used in one situation to new and unfamiliar situations.

At middle school and junior high school, instruction should stress the ability to select from a range of strategies and to create new strategies by combining known techniques.

At middle school and junior high school, instruction should aid in the student’s transition to more abstract reasoning.

Difficulty with paper-and-pencil computational algorithms should not interfere with the learning of problem-solving strategies.

At the junior high school level, calculators should be available so that no student will be excluded from the opportunity to develop these strategies.

All courses in high school mathematics should include some activities in applications.

Teachers of mathematics and teachers of other disciplines should cooperate in assuring that students perceive the relationship of the mathematics they learn to the mathematics applied in problems in those other disciplines.

Qualified mathematics teachers should be used as resource specialists for instructional programs in which mathematics methods are applied in other subjects.

Teachers of mathematics should be prepared in the application of known problem-solving techniques to a variety of problems.

6.6 Special programs stressing problem-solving skills should be devised for special categories of students.

The professionals in mathematics and in special education should work together to identify the process abilities, possible and optimal, for students with handicaps and learning difficulties.

Perception of what mathematical ability encompasses should be broadened beyond the linear, algorithmic stereotype. The significance of spatial perception and spatial relationships in problem solving needs to be stressed, for many students who do not fare well in algorithmic thinking may have special abilities that are spatial and geometric.

Increased attention should be paid to developing the potential of the gifted student of mathematics.

Colleges and schools should cooperate in devising imaginative programs for the mathematically gifted.

In general, programs for the gifted should be based on a sequential program of enrichment through more ingenious problem-solving opportunities rather than through acceleration alone.

Materials and resources of a sophistication and depth suitable to the unusual potential of the gifted student in problem solving should be developed.

Recommendation 7

MATHEMATICS TEACHERS MUST DEMAND OF THEMSELVES AND THEIR COLLEAGUES A HIGH LEVEL OF PROFESSIONALISM

Educators can gain the support of society and the rewards of a truly effective performance by developing, defining, and enforcing professional standards in terms of highly competent professional performance rather than by any other norm. This must be done to provide the nation, its young people, and its future with the mathematics programs worthy of them and of that future.

Within mathematics teaching—
—there are already many well-prepared and effective teachers who provide outstanding professional leadership;
—there are also many teachers who are motivated and desire to improve but who lack adequate preparation and must be given the necessary support to become fully qualified and to improve;

—there are, however, some teachers whose attitudes and functioning are at less than a professional level. In the best interests of students and society, the number of such teachers must be reduced immediately.

Even the best prepared, competent, and dedicated teachers must continue their development to keep abreast of changing needs, tools, and conditions. School administrations have the responsibility to make this possible by providing continuing in-service education and by encouraging teachers to take full advantage of opportunities for maintaining their competence that are offered by professional organizations and universities.

At the beginning of this decade, the schools are faced with a widespread shortage of qualified mathematics teachers. The demand for mathematical competence in many sectors of society is great and growing, and schools find it impossible to compete for individuals who have this desired background. Thus, in many mathematics classrooms, the teacher does not have the subject-matter qualifications for teaching mathematics. School administrations have two obligations: (1) to be forthright and open with the parents about the situation, and (2) to provide special aid and support needed by these teachers until they can make up deficiencies. The professional organizations also have a special responsibility to cooperate in assisting such teachers who are dedicated to improvement. However, the public and its representatives must give high priority to finding ways to solve the worsening problem.

Regardless of preparation, the standard of professionalism should be consistently high, and it is the obligation of any group that wishes to be called a profession to insist that all members maintain this standard.

Teachers must be sensitive to the needs of their students and dedicate themselves to the improvement of student learning as their primary professional objective. The right of students and parents to expect this dedication has correlative responsibilities: a mutual respect and support by the parent of the educational program and a teacher’s professional competence, and the acceptance by the student of ultimate personal and active commitment to his or her own learning.

Any teacher who lacks dedication to these professional ideals and to continued self-improvement should not be retained in teaching. Teachers must accept performance and not protectionism as a synonym for professionalism.

During the decade of the 1980s, the continuing appearance of new concepts and theories in mathematics, in the applications of mathematics, and in the teaching-learning process will affect both curriculum and instruction in school mathematics. In order to remain professional, teachers must continue to study in all three areas. This will require of teachers a new level of motivation and dedication.

## Recommended Actions

### 7.1 Every mathematics teacher should accept responsibility for maintaining teaching competence.

- Full advantage should be taken of all existing opportunities for continuing education.
- Teachers should insist that school districts and colleges make provision for in-service education and staff development opportunities.
- Teachers should belong to professional organizations that are dedicated to the improvement of teaching and learning.
- Teachers should participate actively in the efforts of professional organizations to improve teaching and learning.
- Teachers should share ideas and participate with their peers in cooperative efforts at self-improvement, including observation and constructive criticism of one another.
- Teachers as a profession should insist that all members maintain a consistently high standard of professional behavior. The profession is not obligated to protect those individuals who refuse to live up to reasonable professional standards.
- Bargaining units of organizations representing teachers should include in their requests release time for professional development and attendance at professional conferences that provide in-service education.

### 7.2 School boards and school administrations should take all possible means to assure that mathematics programs are staffed by qualified, competent teachers who remain current in their field.

- Necessary incentives must be found to attract competent and dedicated teachers to the profession.
- The status, compensation, and teaching conditions necessary for the retention of qualified teachers must be dramatically improved.
- School districts must budget adequately and provide incentives for teachers to participate in in-service education pertinent to their immediate needs as they prepare to meet the challenges of the future.
- School district staffs, including teachers, must plan in-service education that is articulated with local colleges and universities as well as with professional organizations such as the NCTM and its state and local affiliates.
- School administrators should encourage teachers to take an active professional role and should permit them to participate, without penalty, in conferences and vital professional work.
- School systems should maintain well-qualified mathematics specialists or supervisors at all levels to help teachers achieve the pro-
7.3 **Teacher education institutions and agencies should develop new programs of preparation to incorporate the problem-solving emphases recommended.**

- Programs in teacher education must be designed to prepare teachers for new levels of performance and professionalism.
- The effective teaching of problem solving requires thorough preparation both in mathematical content and in teaching methods that develop problem-solving ability.
- Colleges and universities must redesign courses and programs to incorporate revisions recommended for the preparation of mathematics teachers.

7.4 **Certification standards for mathematics teaching should be revised and upgraded to incorporate the needs reflected in these recommendations.**

- Professional organizations should participate in defining standards for certification of teachers.
- State and provincial departments of education should involve professional organizations in setting professional standards and defining qualifications.
- Professional organizations—such as the Mathematical Association of America, Conference Board of the Mathematical Sciences, National Council of Supervisors of Mathematics—in cooperation with the NCTM should continually review and update their guidelines for the preparation of teachers of mathematics.

**Recommendation 8**

**PUBLIC SUPPORT FOR MATHEMATICS INSTRUCTION MUST BE RAISED TO A LEVEL COMMENSURATE WITH THE IMPORTANCE OF MATHEMATICAL UNDERSTANDING TO INDIVIDUALS AND SOCIETY**

Solutions to the problems identified in previous sections cannot be achieved solely within the education community but require active participation and support by parental and societal groups.

The mathematics teaching profession recognizes and respects the right of parents and society to hold it accountable for the mathematical competence of children. However, in calling for particular programs of action, parents and society often mistakenly promote activities that are counterproductive to the realization of the goals they support. Communication and cooperation must bridge this gap.

The re dedication of teachers called for in Recommendation 7 will be meaningless or impossible if society is not committed to supporting the professionalism it rightfully demands. Today, teachers' financial incentives are neither commensurate with the responsibilities they carry nor adequate to attract and retain them in a demanding, crucial, and sometimes burdensome profession. Few school systems have truly adequate supervisory and material support for the teacher in the maintenance and improvement of the instructional environment. More and more, teachers feel the lack of parental understanding of the complexity of their task and the lack of parental cooperation and support in their efforts to instruct children. Furthermore, governmental support for improving the quality of mathematics teaching has dwindled.

Essential to the success of the program outlined in these recommendations are (1) the willingness of government and private funding agencies to listen and respond more sensitively to the professionals in elementary and secondary mathematics education, and (2) the reestablishment by such agencies of institutes and other programs for the continuing education and upgrading of teachers.

At present, there are too many unnecessary obstacles to the effective functioning of teacher and student in a true teaching/learning interaction. These include more and more time required for unproductive record keeping; many unmotivated, undisciplined students; a lack of parental support; ambivalence and vacillation in government regulations; shifting societal priorities; the lack of home and school agreement on out-of-school study assignments.

There are also obstacles faced by school administrators and by the policy-making bodies, the school boards, whose ultimate responsibility is the effectiveness of the school district. It is recognized that in many instances confusions of legalisms and court decisions impose constraints on the ability of these people to accomplish what they agree should be done. Society today is troubled with many complexities. All institutions are threatened. These obstacles should not be an excuse for inaction; rather, in the light of pressing student needs it is imperative that they be surmounted.

All these things make the ideal professional role of the teacher almost impossible; they make it difficult to attract and retain teachers of a level of competence necessary to realize the outlined program. The immediate loser is the individual student, who is excluded from full participation in, and contribution to, society. The ultimate loser is society itself.

The public and its representatives need to confront a serious problem that is increasing in magnitude. There are not enough qualified mathematics teachers to fill mathematics classroom positions. Present enrollments in teacher
preparation programs guarantee that the shortage will increase. Mathematics is of critical importance and should be taught by those well qualified and knowledgeable. Such knowledge is not gained in a crash program of short duration. Parents concerned for their children's future and society concerned for its own future should immediately find extraordinary measures to solve this problem.

The professional community and society share a common goal: to bring all citizens to the full realization of their mathematical capacity. This is a complex and delicate task, and it requires the commitment and cooperation of all segments of society, not just the school, parents, and teachers.

Recommended Actions

8.1 Society must provide the incentives that will attract and retain competent, fully prepared, qualified mathematics teachers.

- School districts must provide compensation commensurate with the professionalism and qualifications necessary to achieve our educational goals in mathematics.
- School districts should investigate a variety of incentives and conditions that will stop the drain of qualified mathematics teachers to other, more highly compensated fields of work.
- School districts should assure mathematics teachers a classroom environment and conditions conducive to effective teaching, including a reasonable class size.
- School districts should provide teaching conditions and incentives that will attract dedicated and competent people into the mathematics teaching profession.

8.2 Parents, teachers, and school administrations must establish new and higher standards of cooperation and teamwork toward the common goal of educating each student to his or her highest potential.

- Professionals must respond to calls for maintaining educational standards and must work cooperatively with parents to specify these standards.
- Parents must support the maintenance of agreed-on standards of achievement and discipline.
- Programs in mathematics that take full advantage of home and school cooperation should be systematically developed.
- Parents should enter the process of determining educational goals as partners, with shared participation and responsibility for the accomplishment of those goals.
- Parents should support the teacher's assignment of homework when it is reasonable and clearly related to the educational objectives.

8.3 Government at all levels should operate to facilitate, not dictate, the attainment of goals agreed on cooperatively by the public's representatives and the professionals.

- Government funding agencies should support an emphasis on research and development in applying mathematics to problem solving.
- Teachers should have advisory roles in all decisions of policy and support.
- When legislation affecting education is required, it should be entered into with caution and only after the involvement of educational professionals in formulating and reviewing the mathematical and pedagogical aspects of such legislation.
- Legislation concerning accountability should take into account the multiple factors that determine school achievement.
- Legislators should avoid tendencies to mandate testing as the sole criterion for the evaluation of educational success.
- Mandates for the achievement of minimal competencies should not limit the school mathematics program in its broader range of essential goals.
- Legislation should not determine educational goals but when necessary should facilitate the achievement of cooperatively agreed-on objectives.
- Dual respect and effective articulation must become commonplace between the civic leaders who appropriately call for educational accountability and the educational professionals who must formulate responses.
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