Reshaping School Mathematics:
A Philosophy and Framework for Curriculum

Why?
The governing board of the National Research Council recognized the need for urgent reform in mathematics education and approved the project. The premise of the document, Reshaping School Mathematics: A Philosophy and Framework for Curriculum, was to provide a framework to restructure the mathematics curriculum to meet the needs of American children. According to the document, “What is required is a complete redesign of the content of school mathematics and the way it is taught?” (p. 1).

The report identified several changes affecting mathematics education:
- Changes in the need for mathematics
  - More workers need the ability to apply analytical skills. The increase of data in daily life also requires individuals to have quantitative literacy.
- Changes in mathematics and how it is used
  - In the last 25 years, mathematics has been applied to a wide variety of new problem types.
- Changes in the role of technology
  - The prevalence of calculators and computers raises questions about what mathematics should be taught and how it should be taught.
- Changes in American society
  - Demographic changes as well as workplace changes place extraordinary burdens on mathematics education. In the twenty-first century, more jobs for workers will require mathematics.
- Changes in understandings of how students learn
  - Students learn by connecting new knowledge to prior knowledge. “Learning is not a process of passively absorbing information and storing it in easily retrievable fragments as a result of repeated practice and reinforcement” (p. 3).
- Changes in international competitiveness
  - The global economy influences American society and U.S. students do not compare well in mathematics to students from other countries.

These changes suggest that the mathematics curriculum needs to be changed in order to meet the needs of students in the twenty-first century.

What?
The document emphasizes two issues discussed in Everybody Counts (National Research Council, 1989) and the NCTM Curriculum and Evaluation Standards (National Council of Teachers of Mathematics, 1989):
- “Changing perspectives on the need for mathematics, the nature of mathematics, and the learning of mathematics”
- “Changing roles of calculators and computers in the practice of mathematics” (p. xi).

The purpose of the document was to:
• establish a framework for reform of school mathematics in the United States;
• add to a growing movement in changing mathematics education to aid local reforms;
• provide structure for a national movement.

Who?
The MSEB curriculum framework task force was assembled to write the document.

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What was produced?
The MSEB writing team produced a document that offered six perspectives to improve mathematics education in the United States:

1) A practical philosophy
2) A technological perspective
3) A research perspective
4) A framework for change
5) Goals
6) Enabling conditions

**A practical philosophy**

Improving the mathematics curriculum requires analyzing the nature and goals of mathematics education. The teaching of mathematics serves several goals:

- A practical goal: to help individuals solve problems of everyday life.
- A civic goal: to enable citizens to participate intelligently in civic affairs.
- A professional goal: to prepare students for jobs, vocations, or professions.
- A cultural goal: to impart a major element of human culture. (p. 7)

There are two fundamental questions about school mathematics:

1. **What is mathematics?**
   - Mathematical reasoning is the test of truth. Mathematical models are apt and useful.
2. **What does it mean to know mathematics?**
   - “If mathematics is a science and language of patterns, then to know mathematics is to investigate and express relationships among patterns” (p. 12).

These questions suggest that:

- “Changing the public philosophy of school mathematics is an essential step in effecting reform of mathematics education” (p. 15).
  - The public views mathematics as a fixed set of facts and procedures.
  - The public views mathematics as calculating answers using a given set of procedures.
- A practical test of this philosophy is the effect it has on the teaching of mathematics.

**Technological perspective**

The technological impact on mathematics is urgent as well as controversial.

Current and new computing devices will

- Decrease the value of many manual skills traditionally taught in the school mathematics curriculum;
- Increase the importance of many areas of mathematics that now are rarely taught;
- Focus attention as much on problem formulation as on problem solving;
- Make possible tools for teaching and learning of a sophistication still largely undreamed of by most mathematics educators. (pp. 18-19)

Recommendations about technology suggest that by the year 2000,
• All students should have available hand-held calculators with functionality appropriate to their grade level.
• All mathematics classrooms should have permanently installed contemporary computers with display units conveniently visible by all students.
• There should be sufficient computer facilities available for laboratory and out-of-classroom needs of all students. (p. 21)

Curriculum issues related to technology:
• Procedural and conceptual knowledge
  o “What level of manipulative skill is necessary in order to be able to understand — and then use — mathematics in a problem solving context?” (p. 26)
• Transfer of knowledge
  o “How can school instruction provide students with a background that will enable them to apply what they have learned in out-of-school contexts?” (p. 26)
• Instructional uses of technology
  o “Technological research has just begun to create tools with the power to alter significantly the traditional process of instruction. What kinds of mathematics comprehension can these new tools foster?” (p. 26)

Research perspective
• Learning mathematics
  o Research suggests that higher-order thinking develops from less structured and indirect teaching, rather than solely from teacher presentations of content.
  o Cooperative learning strategies can enhance learning as can teaching of cognitive and metacognitive strategies.
• Constructed knowledge
  o Children interpret and construct knowledge, rather than just absorb it.
• Procedural knowledge
  o Paper-and-pencil calculations alone do not aid in conceptual understanding, so there is a recommendation to reduce emphasis on procedures.
• Mastery of subject matter
  o Research has only just begun developing maps to describe the phases through which children progress as they build understanding.
  o Research has to determine when rules should come first and when they should not.
• Problem solving
  o Strategies for problem solving can be effectively taught.
• Making sense of mathematics
  o Genuine applications of mathematics are not taught well in the current curriculum.

(pp. 27-33)

Framework for Change: Fundamental Principles
• Mathematics education must focus on the development of mathematical power.
• Calculators and computers should be used throughout the mathematics curriculum.
• Relevant applications should be an integral part of the curriculum.
• Each part of the curriculum should be justified on its own merits.
• Curricular choices should be consistent with contemporary standards for school mathematics.
• Mathematics instruction at all levels should foster active student involvement. (pp. 36-39)

Goals for Curricular Change
• A primary goal of elementary school mathematics should be to develop number sense.
• Elementary school mathematics should provide an effective foundation for all aspects of mathematics.
• Middle school mathematics should emphasize the practical power of mathematics.
• A major goal of the secondary mathematics curriculum should be to develop symbol sense.
• Secondary school mathematics should introduce the entire spectrum of mathematical sciences.
• Calculators should be available in all instructional and assessment situations.
• Students learning mathematics should use real objects and real data.
• Mathematics in school should reinforce other school subjects and vice versa.
• Students should apprehend that in mathematics, reasoning is the standard of truth.
• All students should study mathematics every year they are in school. (pp. 41-47)

Enabling Conditions for Improving Mathematics Education
• Professional development is needed for a reform curriculum to be effective.
  o “As teachers implement important, timely and exciting changes, they will require continuing programs of professional support” (p. 48).
• Instructional materials determine what is taught and how it is taught in the classroom.
  o Teachers “seldom give significant attention to topics not included in the text” (p. 49).
• Assessment with today’s standardized tests drives the curriculum.
  o These tests “stray far from both the available and the adopted curriculum; none even gets near the ideal curriculum” (p. 50).

Future Recommendations
The MSEB writing teams also made predictions about what mathematics education should look like in the twenty-first century. They indicated that mathematics education will include:
• Greater breadth of mathematical sciences.
• More students who take more mathematics courses.
• An increase in the use of technology in the classroom.
• Enhanced professionalism for teachers.
• Increased need for higher-order thinking skills.
• More sophisticated means of assessment. (p. 51)

References
